

The Fate of the Sound and Diffusion in Holographic Magnetic Field

Evgeny I. Buchbinder¹, Alex Buchel^{1,2}

¹*Perimeter Institute for Theoretical Physics*

Waterloo, Ontario N2J 2W9, Canada

²*Department of Applied Mathematics*

University of Western Ontario

London, Ontario N6A 5B7, Canada

Abstract

It was shown in [1] that in the presence of the magnetic field the sound waves in (2+1) dimensional plasma disappear and are replaced by a diffusive mode. Similarly, the shear and charge diffusion fluctuations form a subdiffusive mode. However, since the limit of small magnetic field does not commute with the hydrodynamic limit it is not obvious whether or not these modes are stable under higher order corrections. Using AdS/CFT correspondence we show that in the case of the $M2$ -brane plasma these modes do exist as we find the corresponding supergravity solutions. This allowed us to compute the conductivity and the shear viscosity to all orders in magnetic field. We find that the viscosity to entropy ratio saturates the Kovtun-Son-Starinets bound. This extends the universality property of the shear viscosity to the case of the strongly coupled plasma in external magnetic field.

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1 Introduction

In [1] we studied first-order viscous magneto-hydrodynamics of strongly coupled (2+1)-dimensional conformal systems in the framework of gauge theory/string theory correspondence of Maldacena [2]. The existence of essentially soluble holographic model, *i.e.*, the $M2$ -brane plasma [2, 3, 4], allows one to probe intricate aspects of strongly coupled relativistic conformal viscous fluids in the presence of external magnetic field. In particular, it is known that the hydrodynamic limit in (2+1) dimensions does not commute with the limit of small magnetic field [5]. As a result, one expects a drastic modification of the transport properties of magnetized fluids. Indeed, in [1] it was found that the sound wave in magnetic plasma can propagate only in the limit of vanishing magnetic field. Depending on the scaling of the magnetic field in the hydrodynamic limit either only the attenuation or both the attenuation and the speed of the sound waves are effected by the background magnetic field. In the latter case, it was found that the magnetic field reduces the effective speed of propagating sound modes, while enhancing their attenuation. The field theoretical arguments in the setting of Hartnoll-Kovtun-Müller-Sachdev (HKMS) magneto-hydrodynamics [5] further suggest that a hydrodynamic mode with linear dispersion relation disappears from the spectrum for finite magnetic field. Similarly, in this regime, the standard diffusive modes in viscous fluids become subdiffusive, with $\omega \propto -iq^4$ dispersion. We would like to test these predictions in the holographic model of magneto-hydrodynamics of $M2$ -brane plasma.

Our second motivation is to understand how background fields affect the viscosity of strongly coupled fluids. Previously, it was discovered that relativistic holographic plasma fluids (with various gauge groups, matter content, with or without chemical potentials for conserved $U(1)$ charges, with non-commutative spatial directions) have a universal value of the shear viscosity at infinite 't Hooft coupling [6, 7, 8, 9, 10, 11]. The universality of the ratio of the shear viscosity to the entropy density extends also to non-relativistic holographic CFTs [12, 13]. Additionally, four-dimensional conformal CFTs with a dual holographic description and equal central charges $a = c$ have a universal leading finite 't Hooft coupling correction [14, 15]. On the contrary, the leading non-planar correction to the ratio of shear viscosity to the entropy density is not universal [16]. Since in magnetized (2+1) fluids the shear mode becomes subdiffusive, and the dual holographic setting falls outside the most general universality class considered in

[9], one naturally questions whether the ratio of shear viscosity to the entropy density continues to be universal.

The paper is organized as follows. In the next section, we review magneto-hydrodynamics of HKMS and its soluble holographic realization as hydrodynamics of dyonic black holes in M-theory. In section 3, we extend the supergravity analysis of [1] and discuss propagation of 'sound waves' in $M2$ -brane plasma in the hydrodynamic limit with finite external magnetic field. In section 4, we study 'shear modes' and compute the ratio of the shear viscosity to the entropy density of the $M2$ -brane plasma in the external magnetic field. In section 5, we comment on a computation of the shear viscosity using the Kubo formula. Some technical details are presented in Appendix A.

After this work was completed, the paper [17] appeared which approaches magneto-hydrodynamics from the gravity side along the lines of [18]. In the future, it would be interesting to compare the results of [17] with the HKMS approach.

2 Magneto-Hydrodynamics and Dyonic Black Hole Geometry

2.1 Hydrodynamic Modes in the Presence of Magnetic Field

In this section, we will review magneto-hydrodynamics in (2+1) dimensions following [1]. We are interested in hydrodynamic properties of the (2+1) dimensional theory on the large number of $M2$ -branes in the presence of the external magnetic field. This theory can be understood as the maximally supersymmetric gauge theory in three dimensions at the infrared fixed point. The equations of motion and the conformal properties can be rigorously derived using the fact that the theory admits a holographic dual description as M-theory on $AdS_4 \times S^7$ [2, 3, 4]. The appropriate field theory equations then follow from the symmetries of the AdS background. This was studied in detail in [1] and here we will quote the results. The relevant field theory equations of motion are just the conservation laws of the form

$$\begin{aligned}\partial^\nu T_{\mu\nu} &= F_{\mu\nu} J^\nu, \\ \partial_\mu J^\mu &= 0,\end{aligned}\tag{2.1}$$

where $T_{\mu\nu}$ is the stress-energy tensor, J^μ is the current and $F^{\mu\nu}$ is the *external* electro-magnetic field. In the present paper, it is taken to be magnetic, that is

$$F_{0i} = 0, \quad i = 1, 2, \quad F_{ij} = \epsilon_{ij} B.\tag{2.2}$$

Another important equation is

$$T^\mu{}_\mu = 0, \quad (2.3)$$

which means that the field theory under study is conformal. Eq. (2.3) is the consequence of the fact that the leading near-the-boundary asymptotics of the gauge field in AdS is constant, which implies that the magnetic field represents a marginal deformation. See [1] for details.

The expressions for $T^{\mu\nu}$ and J^ν to first order in derivatives were derived in [5] from postulating positivity of the entropy production along the lines of Landau and Lifshitz [19]. The stress-energy tensor is given by the standard expression

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P \Delta^{\mu\nu} - \eta (\Delta^{\mu\alpha} \Delta^{\nu\beta} (\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \Delta^{\mu\nu} \partial_\gamma u^\gamma) - \zeta \Delta^{\mu\nu} \partial_\alpha u^\alpha. \quad (2.4)$$

Here

$$\Delta^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu, \quad (2.5)$$

u^μ is the fluid 3-velocity, ϵ and P are the energy density and the pressure respectively, and η and ζ are the shear and bulk viscosity. Since our theory is conformal it follows that $\zeta = 0$. It is important to note that P is different from the thermodynamic pressure p [5],

$$P = p - MB, \quad (2.6)$$

where M is the magnetization. Also note that conformal invariance implies that

$$c_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{1}{2}. \quad (2.7)$$

Similarly, the current J^μ is given by

$$J^\mu = \rho u^\mu + \sigma_Q \Delta^{\mu\nu} (-\partial_\nu \mu + F_{\nu\alpha} u^\alpha + \frac{\mu}{T} \partial_\nu T), \quad (2.8)$$

where ρ is the charge density, μ is the chemical potential, T is the temperature and σ_Q is the conductivity coefficient. To study fluctuations around the equilibrium state

$$u^\mu = (1, 0, 0), \quad T = \text{const.}, \quad \mu = \text{const.}, \quad (2.9)$$

we choose $(\delta u_1 = \delta u_x, \delta u_2 = \delta u_y, \delta T, \delta \mu)$ as the independent quantities. As usual, all the fluctuations are of the plane-wave form $\exp(-i\omega t + i q y)$. In this paper, we are interested in hydrodynamics with no net charge density and, correspondingly, with no chemical potential

$$\rho = 0, \quad \mu = 0. \quad (2.10)$$

In this case, as was shown in [1], the equations for the linear fluctuations get separated into the two decoupled pairs. The first pair reads

$$\begin{aligned}\omega \left(\frac{\partial \epsilon}{\partial T} \right)_\mu \delta T - q(\epsilon + P) \delta u_y &= 0, \\ \omega(\epsilon + P) \delta u_y - q \left(\frac{\partial P}{\partial T} \right)_\mu \delta T + iq^2 \eta \delta u_y + i\sigma_Q B^2 \delta u_y &= 0.\end{aligned}\tag{2.11}$$

If we set $B = 0$ these equations describe the sound waves with dispersion relation

$$\omega = \pm \frac{q}{2} - iq^2 \frac{\eta}{\epsilon + P},\tag{2.12}$$

where eq. (2.7) has been used. We will refer to these equations as to the “sound channel”. The second decoupled pair of equations is

$$\begin{aligned}\omega(\epsilon + P) \delta u_x - qB\sigma_Q \delta \mu + i\sigma_Q B^2 \delta u_x + iq^2 \eta \delta u_x &= 0, \\ \omega \left(\frac{\partial \rho}{\partial \mu} \right)_T \delta \mu + q\sigma_Q B \delta u_x + iq^2 \sigma_Q \delta \mu &= 0.\end{aligned}\tag{2.13}$$

Note that here we are assuming that the susceptibility $\left(\frac{\partial \rho}{\partial \mu} \right)_T$ does not vanish at $\rho = \mu = 0$. If we set $B = 0$ these two equations further decouple. One equation describes the shear mode δu_x with dispersion relation

$$\omega = -iq^2 \frac{\eta}{\epsilon + P}.\tag{2.14}$$

The other one describes the charge diffusion mode $\delta \mu$ with dispersion relation

$$\omega = -iq^2 \frac{\sigma_Q}{\left(\frac{\partial \rho}{\partial \mu} \right)_T}.\tag{2.15}$$

We will refer to eqs. (2.13) as to the “shear channel”. If we turn on the magnetic field B , the hydrodynamic modes undergo a drastic change. The reason, as one can see from eqs. (2.11) and (2.13), is that the limit of small B does not commute with the hydrodynamic limit of small ω and q . So the magnetic field cannot be thought of as a small perturbation. In the case of non-zero B (kept fixed in the hydrodynamic limit) we obtain the following solutions. In the sound channel we do not get the sound waves anymore. Instead, we obtain a constant solution

$$\omega = -i\sigma_Q \frac{B^2}{\epsilon + P},\tag{2.16}$$

and a diffusive mode

$$\omega = -\frac{iq^2}{2} \frac{\epsilon + P}{\sigma_Q B^2}. \quad (2.17)$$

One can interpret (2.17) as that the effective speed of sound vanishes once the magnetic field is turned on. In the shear channel, the usual shear and diffusive modes disappear. Instead, we also obtain a constant solution (2.16) and a subdiffusive mode

$$\omega = -iq^4 \frac{\eta}{B^2 \left(\frac{\partial \rho}{\partial \mu} \right)_T}. \quad (2.18)$$

The modes (2.17) and (2.18) will be the main focus of our paper.

Since the limit of small B does not commute with the hydrodynamic limit one can worry that the solutions (2.17) and (2.18) cannot be trusted. Indeed, these solutions imply a hierarchy of amplitudes. From eqs. (2.11) and (2.17) it follows that

$$\frac{\delta T}{\delta u_y} \sim \frac{1}{q}, \quad (2.19)$$

and from eqs. (2.13) and (2.18) it follows that

$$\frac{\delta \mu}{\delta u_x} \sim \frac{1}{q}. \quad (2.20)$$

Hence, given the amplitudes of the linearized fluctuations δu_x and δu_y , we find that the amplitudes of the linearized fluctuations δT and $\delta \mu$ are strongly enhanced in the hydrodynamic limit. Then one can expect that the solutions (2.17) and (2.18) are unstable under higher order (higher derivative) corrections to $T^{\mu\nu}$ and J^μ . That is, terms which are naively of higher order because they are suppressed by higher powers of ω and q can, in fact, modify hydrodynamic equations at lower order because they come with a large amplitude.

Unfortunately, at the level of the effective field theory it is very difficult to answer whether or not these solutions exist. However, for the case of the $M2$ -brane plasma we can use the description in terms UV complete M-theory on $AdS_4 \times S^7$ background. Our gravitational analysis in the later sections will show that in the case of the $M2$ -brane plasma with large number of $M2$ -branes the modes (2.17) and (2.18) do exist. Moreover, finding these solutions on the gravity side will allow us to calculate the conductivity coefficient σ_Q and the shear viscosity η to all orders in magnetic field. These results indicate that AdS/CFT correspondence is a helpful method to study hydrodynamic modes whose very existence is subtle from the field theory prospective.

2.2 Supergravity Magneto-Hydrodynamics

According to AdS/CFT correspondence, in the limit when the number of $M2$ -branes becomes very large, their dynamics can be described by the eleven-dimensional supergravity on $AdS_4 \times S^7$. For our purposes, this theory can be consistently truncated to Einstein-Maxwell theory on AdS_4 [20]. The supergravity action is then given by¹

$$S = \frac{1}{g^2} \int d^4x \sqrt{-g} \left[-\frac{1}{4}R + \frac{1}{4}F_{MN}F^{MN} - \frac{3}{2} \right], \quad (2.21)$$

where the bulk coupling constant g is given by

$$\frac{1}{g^2} = \frac{\sqrt{2}N^{3/2}}{6\pi}, \quad (2.22)$$

where N is the number of $M2$ -branes. The corresponding equations of motion are

$$\begin{aligned} R_{MN} &= 2F_{ML}F_N^L - \frac{1}{2}g_{MN}F_{LP}F^{LP} - 3g_{MN}, \\ \nabla_M F^{MN} &= 0. \end{aligned} \quad (2.23)$$

The equilibrium state of magneto-hydrodynamics is described by (asymptotically AdS_4) dyonic black hole geometry with planar horizon whose Hawking temperature is identified with the plasma temperature. The solution looks as follows [21]

$$\begin{aligned} ds^2 &= -c_1(r)^2 dt^2 + c_2(r)^2(dx^2 + dy^2) + c_3(r)^2 dr^2, \\ F &= h\alpha^2 dx \wedge dy + q\alpha dr \wedge dt, \end{aligned} \quad (2.24)$$

where

$$c_1(r)^2 = \frac{\alpha^2}{r^2}f(r), \quad c_2(r)^2 = \frac{\alpha^2}{r^2}, \quad c_3(r)^2 = \frac{\alpha^2}{f(r)r^2}, \quad (2.25)$$

and

$$f(r) = 1 + (h^2 + q^2)r^4 - (1 + h^2 + q^2)r^3. \quad (2.26)$$

In these coordinates, $r = 1$ corresponds to the horizon and $r = 0$ is the boundary. The black hole parameters (h, q, α) are related to the field theory magnetic field, chemical potential and temperature as [21]

$$B = h\alpha^2, \quad \mu = -q\alpha, \quad T = \frac{\alpha}{4\pi}(3 - h^2 - q^2). \quad (2.27)$$

¹For simplicity, we set the radius of AdS_4 to unity.

Note that the xy -component of F goes to a constant $h\alpha^2$ on the boundary and is identified with the boundary theory magnetic field B . On the other hand, the t -component of the vector potential behaves near the boundary as $A_t = -q\alpha r$. It is interpreted as the chemical potential in the boundary theory.

Now we list some thermodynamic properties of the dyonic black hole. See [21] for more details. The appropriate thermodynamic potential is obtained by evaluating the (renormalized) action (2.21) and is given by

$$\Omega = -pV = V \frac{1}{g^2} \frac{\alpha^3}{4} \left(-1 - \frac{\mu^2}{\alpha^2} + 3 \frac{B^2}{\alpha^4} \right), \quad (2.28)$$

where V is the area of the (x, y) -plane and p is the thermodynamic pressure. The other quantities of importance are the density of energy, entropy and electric charge. They are given by

$$\epsilon = \frac{1}{g^2} \frac{\alpha^3}{2} \left(1 + \frac{\mu^2}{\alpha^2} + \frac{B^2}{\alpha^4} \right), \quad (2.29)$$

$$s = \frac{\pi}{g^2} \alpha^2, \quad (2.30)$$

and

$$\rho = \frac{1}{g^2} \alpha \mu. \quad (2.31)$$

In addition, we introduce magnetization per unite area

$$M = -\frac{1}{V} \left(\frac{\partial \Omega}{\partial B} \right)_{T, \mu} = -\frac{1}{g^2} \frac{B}{\alpha}. \quad (2.32)$$

Just like on the field theory side, we introduce

$$P = p - MB. \quad (2.33)$$

One can show [21] that it is P rather than p that coincides with the spatial components $\langle T^{xx} \rangle$ and $\langle T^{yy} \rangle$ of the stress-energy tensor, just like we have in eq. (2.4). It is straightforward to check that

$$P = \frac{\epsilon}{2}, \quad (2.34)$$

which is consistent with conformal invariance. In this paper, we consider magneto-hydrodynamics in the absence of the net charge density ρ . Thus, we set $q = 0$. Note that even though ρ and μ vanish the derivative

$$\left(\frac{\partial \rho}{\partial \mu} \right)_T = \frac{\alpha}{g^2}. \quad (2.35)$$

is non-zero.

To study a holographic dual of the hydrodynamic modes, we need to find linear fluctuations of the supergravity equations of motion (2.23) around the black hole background (2.24),

$$\begin{aligned} g_{MN} &\rightarrow g_{MN} + h_{MN} , \\ A_M &\rightarrow A_M + a_M . \end{aligned} \tag{2.36}$$

It is convenient to impose the gauge

$$h_{tr} = h_{xr} = h_{yr} = h_{rr} = 0 , \quad a_r = 0 . \tag{2.37}$$

In parallel with field theory, the fluctuations g_{MN} and a_M will be of the form $\exp(-i\omega t + i q y)$ and the r -dependence is to be obtained from solving the linearized Einstein and Maxwell equations (2.23). As was explained in [1], for both q and h non-zero, all the metric and gauge field fluctuations couple to each other and no decoupling of various modes exist. The reason is that the background (2.24) does not have any symmetry which usually allows one to decouple scalar-, vector- and tensor-type fluctuations. However, in the case of interest $q = 0$ there exist two sets of decoupled fluctuations. The first set corresponds to the field theory sound channel. The corresponding fluctuations are

$$\{h_{tt}, h_{ty}, h_{xx}, h_{yy}, a_x\} . \tag{2.38}$$

The second set corresponds to the field theory shear channel and includes the following fluctuations

$$\{h_{tx}, h_{xy}, a_t, a_y\} . \tag{2.39}$$

In the next section, we will show that the fluctuations (2.38) indeed correctly describe the diffusive mode (2.17). In section 4, we will show that the fluctuations (2.39) indeed describe the subdiffusive mode (2.18).

3 The Fate of the Sound Waves

In this section, we will consider the equations of motion for the fluctuations (2.38). Let us introduce

$$\begin{aligned}
h_{tt} &= c_1(r)^2 \hat{h}_{tt} = e^{-i\omega t + i q y} c_1(r)^2 H_{tt} , \\
h_{ty} &= c_2(r)^2 \hat{h}_{ty} = e^{-i\omega t + i q y} c_2(r)^2 H_{ty} , \\
h_{xx} &= c_2(r)^2 \hat{h}_{xx} = e^{-i\omega t + i q y} c_2(r)^2 H_{xx} , \\
h_{yy} &= c_2(r)^2 \hat{h}_{yy} = e^{-i\omega t + i q y} c_2(r)^2 H_{yy} , \\
a_x &= i e^{-i\omega t + i q y} \hat{a}_x ,
\end{aligned} \tag{3.1}$$

where H_{tt} , H_{ty} , H_{xx} , H_{yy} and, \hat{a}_x are functions of the radial coordinate only and $c_1(r)$ and $c_2(r)$ are defined in eqs. (2.25) and (2.26). Expanding eqs. (2.23) to linear order we obtain the following system of equations

$$\begin{aligned}
0 = & H_{tt}'' + H_{tt}' \left[\ln \frac{c_1^2 c_2}{c_3} \right]' + \frac{1}{2} [H_{xx} + H_{yy}]' \left[\ln \frac{c_2}{c_1} \right]' - \frac{c_3^2}{2c_1^2} \left(q^2 \frac{c_1^2}{c_2^2} (H_{tt} + H_{xx}) \right. \\
& \left. + \omega^2 (H_{xx} + H_{yy}) + 2\omega q H_{ty} \right) - 3 \frac{c_3^2}{c_2^4} h^2 \alpha^4 (H_{xx} + H_{yy}) + 6 \frac{c_3^2}{c_2^4} h \alpha^2 q \hat{a}_x ,
\end{aligned} \tag{3.2}$$

$$0 = H_{ty}'' + H_{ty}' \left[\ln \frac{c_2^4}{c_1 c_3} \right]' + \frac{c_3^2}{c_2^2} \omega q H_{xx} - 4 \frac{c_3^2}{c_2^4} h \alpha^2 (h \alpha^2 H_{ty} + \omega \hat{a}_x) , \tag{3.3}$$

$$\begin{aligned}
0 = & H_{xx}'' + \frac{1}{2} H_{xx}' \left[\ln \frac{c_1^5 c_2}{c_3^2} \right]' + \frac{1}{2} H_{yy}' \left[\ln \frac{c_2}{c_1} \right]' + \frac{c_3^2}{2c_1^2} \left(\omega^2 (H_{xx} - H_{yy}) - q^2 \frac{c_1^2}{c_2^2} (H_{tt} + H_{xx}) \right. \\
& \left. - 2\omega q H_{ty} \right) - \frac{c_3^2}{c_2^4} h^2 \alpha^4 (H_{xx} + H_{yy}) + 2 \frac{c_3^2}{c_2^4} h \alpha^2 q \hat{a}_x ,
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
0 = & H_{yy}'' + \frac{1}{2} H_{yy}' \left[\ln \frac{c_1^5 c_2}{c_3^2} \right]' + \frac{1}{2} H_{xx}' \left[\ln \frac{c_2}{c_1} \right]' + \frac{c_3^2}{2c_1^2} \left(\omega^2 (H_{yy} - H_{xx}) + q^2 \frac{c_1^2}{c_2^2} (H_{tt} - H_{xx}) \right. \\
& \left. + 2\omega q H_{ty} \right) - \frac{c_3^2}{c_2^4} h^2 \alpha^4 (H_{xx} + H_{yy}) + 2 \frac{c_3^2}{c_2^4} h \alpha^2 q \hat{a}_x ,
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
0 = & \hat{a}_x'' + \hat{a}_x' \left[\ln \frac{c_1}{c_3} \right]' + \frac{c_3^2}{c_1^2} \hat{a}_x \left(\omega^2 - \frac{c_1^2}{c_2^2} q^2 \right) + \frac{c_3^2}{2c_2^2} h \alpha^2 \left(q (H_{tt} + H_{xx} + H_{yy}) \right. \\
& \left. + 2\omega \frac{c_2^2}{c_1^2} H_{ty} \right) .
\end{aligned} \tag{3.6}$$

In addition, we obtain three first class constraints from varying the action with respect to the gauge fixed metric components h_{tr} , h_{yr} and h_{rr}

$$0 = \omega \left([H_{xx} + H_{yy}]' + \left[\ln \frac{c_2}{c_1} \right]' (H_{xx} + H_{yy}) \right) + q \left(H'_{ty} + 2 \left[\ln \frac{c_2}{c_1} \right]' H_{ty} \right), \quad (3.7)$$

$$0 = q \left([H_{tt} - H_{xx}]' - \left[\ln \frac{c_2}{c_1} \right]' H_{tt} \right) + \frac{c_2^2}{c_1^2} \omega H'_{ty} + 4 h \alpha^2 \frac{\hat{a}'_x}{c_2^2}, \quad (3.8)$$

$$0 = [\ln c_1 c_2]' [H_{xx} + H_{yy}]' - [\ln c_2^2]' H'_{tt} + \frac{c_3^2}{c_1^2} \left(\omega^2 (H_{xx} + H_{yy}) + 2\omega q H_{ty} \right. \\ \left. + q^2 \frac{c_1^2}{c_2^2} (H_{tt} - H_{xx}) \right) - 2 \frac{c_3^2}{c_2^4} h^2 \alpha^4 (H_{xx} + H_{yy}) + 4 \frac{c_3^2}{c_2^4} h \alpha^2 q \hat{a}_x. \quad (3.9)$$

If we set in these equations $h = 0$, we see that we can also consistently set $\hat{a}_x = 0$. Then the remaining equations for H_{tt} , H_{ty} , H_{xx} and H_{yy} can be shown to coincide with those in [22] and describe the sound waves with dispersion relation (2.12). See [22] for details. To continue, we note that the gauge (2.37) does not fully fix the diffeomorphism invariance. Since we have five (second order in derivatives) equations and three (first order in derivatives) constraints, there are precisely two combinations invariant under the residual gauge transformations. They were found in [1] to be

$$Z_H = 4 \frac{q}{\omega} H_{ty} + 2 H_{yy} - 2 H_{xx} \left(1 - \frac{q^2 c'_1 c_1}{\omega^2 c'_2 c_2} \right) + 2 \frac{q^2 c_1^2}{\omega^2 c_2^2} H_{tt}, \\ Z_A = \hat{a}_x + \frac{1}{2q} h \alpha^2 (H_{xx} - H_{yy}). \quad (3.10)$$

Then from eqs. (3.2)-(3.6) and (3.7)-(3.9) we obtain two decoupled gauge invariant equations for Z_H and Z_A

$$0 = A_H Z_H'' + B_H Z_H' + C_H Z_H + D_H Z_A' + E_H Z_A, \quad (3.11)$$

$$0 = A_A Z_A'' + B_A Z_A' + C_A Z_A + D_A Z_H' + E_A Z_H. \quad (3.12)$$

The connection coefficients $\{A_H, \dots, E_A\}$ can be computed from eqs. (3.2)-(3.9) and (3.10) using explicit expressions for the c_i 's in (2.25). Since these coefficients are very long and cumbersome we will not present them in the paper. Below we will present these equations in the limit of small ω and q .

As the next step, we will discuss the boundary conditions. According to the general prescription [23, 24], in order to obtain the dispersion relation (poles in the retarded Green's function) we have to impose the following boundary conditions

- Z_H and Z_A are incoming waves at the horizon $r = 1$.
- Z_H and Z_A satisfy the Dirichlet boundary conditions on the boundary $r = 0$. That is, both Z_H and Z_A have to vanish at $r = 0$.

In [1] it was shown that Z_H and Z_A have the following behavior at the horizon

$$\begin{aligned} Z_H(r) &= f(r)^{-i\mathfrak{w}/2} z_H(r), \\ Z_A(r) &= f(r)^{-i\mathfrak{w}/2} z_A(r), \end{aligned} \tag{3.13}$$

where we introduce

$$\mathfrak{w} = \frac{\omega}{2\pi T}, \quad \mathfrak{q} = \frac{q}{2\pi T}. \tag{3.14}$$

The functions z_H and z_A are now regular and non-vanishing at the horizon. In addition, they have to satisfy the Dirichlet boundary conditions at $r = 0$.

A crucial ingredient in search for the solution is a proper understanding of the correct relative normalization. Since our equations (3.11) and (3.12) are homogeneous, we can normalize one of the functions, say z_H , to be unity at the horizon $r = 1$. However, after that we cannot normalize z_A . Moreover, the ratio $\frac{z_A}{z_H}$ can depend on q . Since we are going to solve the equations of motion perturbatively in q it is important to establish how $\frac{z_A}{z_H}$ scales with q . We recall that we are looking for a diffusive solution with $\omega \sim q^2$. Then from eq. (3.10) it follows that for small q

$$z_H \sim \frac{1}{q^2} H_{tt}, \quad z_A \sim \hat{a}_x. \tag{3.15}$$

Note that due to an additional symmetry between x and y at $q = 0$ it follows that $H_{xx} = H_{yy}$ at $q = 0$ and the leading term for small q in z_A is \hat{a}_x . Furthermore, we know that H_{tt} is dual to the tt -component of the boundary stress-energy tensor whereas \hat{a}_x is dual to the x -component of the boundary current J^x . Going back to the field theory side, using eqs. (2.4) and (2.8) one can show that

$$\frac{\delta J^x}{\delta T^{tt}} \sim \frac{\delta u_y}{\delta T} \sim q, \tag{3.16}$$

where eq. (2.19) has been used. Then we find that $\frac{z_A}{z_H} \sim q^3$. Let us now parameterize the ansatz for our solution. We parameterize the dispersion relation as follows

$$\mathfrak{w} = -i \frac{\mathfrak{q}^2}{h^2} C. \tag{3.17}$$

The additional h^2 in the denominator is dictated by the field theory result (2.17). The coefficient C is now assumed to have a perturbative expansion in h . Similarly, it is

convenient to pull out the appropriate powers in h in the \mathfrak{q} -expansion of Z_H and Z_A . We end up with the following ansatz

$$\begin{aligned} Z_H &= f(r)^{-i\mathfrak{w}/2} \left(F_1(r) + \frac{\mathfrak{q}^2}{h^2} F_3(r) + \mathcal{O}(\mathfrak{q}^4) \right), \\ Z_A &= f(r)^{-i\mathfrak{w}/2} \frac{\mathfrak{q}^3}{h^5} (F_2(r) + \mathcal{O}(\mathfrak{q}^2)) . \end{aligned} \quad (3.18)$$

One can show that with these powers of h in the denominators, the functions $F_i(r)$ have now a perturbative expansion in h^2 . Note that since z_H is normalized to be unity at the horizon, we can choose $F_1(r) = 1$ and $F_3(r) = 0$ at $r = 1$.

First, we will solve eqs. (3.11) and (3.12) to leading order in h , that is ignoring the h -dependence in C and $F_i(r)$. Doing this we will determine that C can be fixed entirely by imposing the proper boundary condition on the fluctuations Z_H and Z_A near the horizon. Thus, we will be able to generalize the procedure and find an analytic expression for C to all orders in h without actually finding the full analytic solution for the fluctuations. We start with the equations to the leading order in \mathfrak{q} . From eq. (3.11) we obtain

$$F_1'' - \frac{2}{r} F_1' = 0 . \quad (3.19)$$

The solution with the prescribed above boundary conditions is

$$F_1(r) = r^3 . \quad (3.20)$$

From eq. (3.21) we get

$$F_2'' - \frac{2+r^3}{r(1-r^3)} F_2' + \frac{3\alpha C^2}{8r(1-r^3)} = 0 , \quad (3.21)$$

where the solution for F_1 (3.20) have been used. The general solution to this equation is given by

$$\begin{aligned} F_2(r) &= C_1 + C_2 \ln(1-r^3) \\ &+ \frac{\alpha C^2}{32} \left[2\sqrt{3} \arctan\left(\frac{1+2r}{\sqrt{3}}\right) - 2\ln(1-r) + \ln(1+r+r^2) \right] . \end{aligned} \quad (3.22)$$

The integration constant C_2 has to be fixed by requiring that $F_2(r)$ is regular at the horizon. It gives

$$C_2 = \frac{\alpha C^2}{16} . \quad (3.23)$$

The other integration constant C_1 is fixed by requiring that $F_2(r)$ vanishes at $r = 0$ to be

$$C_1 = -\frac{\alpha\pi C^2}{32\sqrt{3}}. \quad (3.24)$$

Note that we are not able to fix the diffusion constant C working at leading order in \mathfrak{q} . We have to go to next-to-leading order in \mathfrak{q} and consider the equation for F_3 ²

$$F_3'' - \frac{2}{r}F_3' + \frac{r^4(81(-4 + r^3) - 432C(-2 + r^3) + 512C^2r(-1 + r^3))}{96(1 - r^3)^2} = 0, \quad (3.25)$$

where the above solutions for $F_1(r)$ and $F_2(r)$ have been used. It is possible to find the general solution for $F_3(r)$. We will not write it here because it is rather lengthy. The solution has a logarithmic singularity at the horizon of the form

$$F_3(r) \sim \frac{1}{32}(-9 + 16C) \ln(1 - r). \quad (3.26)$$

Requiring that the solution is smooth fixes C to be

$$C = \frac{9}{16} + \mathcal{O}(h). \quad (3.27)$$

The two integration constants in the solution for $F_3(r)$ are fixed by requiring that $F_3(r)$ vanishes at $r = 0$ and $r = 1$.

From this procedure it becomes clear that to obtain C we need to understand the near horizon structure of the solution and require that it is smooth. It is possible to perform this analysis for arbitrary h . The details are presented in Appendix A. This allows us to find the exact value of C

$$C = \frac{3}{16}(3 - h^2)(1 + h^2). \quad (3.28)$$

Thus, we have managed to reproduce the diffusive mode in the sound channel on the gravity side! We proved that this mode does exist and the exact (to all orders in h) value of the diffusion constant is given by eq. (3.28).

To summarize the results, we have obtained a supergravity solution with the following dispersion relation

$$\mathfrak{w} = -\frac{i\mathfrak{q}^2}{h^2} \frac{3}{16}(3 - h^2)(1 + h^2) + \mathcal{O}(\mathfrak{q}^4). \quad (3.29)$$

²One can worry that going to next-to-leading order in \mathfrak{q} cannot fix C because the equation for F_3 will also depend on higher order coefficients in the dispersion relation. However, it is straightforward to check that it is not the case.

Let us compare it with the field theory counterpart (2.17). For this we will rewrite (2.17) in terms of $(\mathfrak{w}, \mathfrak{q}, \alpha, h)$. From eqs. (3.14), (2.27) and (2.29) it follows that (2.17) can be written as

$$\mathfrak{w} = -\frac{i\mathfrak{q}^2}{h^2} \frac{3}{16} (3 - h^2)(1 + h^2) \frac{1}{g^2 \sigma_Q}. \quad (3.30)$$

Comparing (3.30) and (3.29) we obtain the following answer for the conductivity coefficient

$$\sigma_Q = \frac{1}{g^2} = \frac{\sqrt{2} N^{3/2}}{6\pi}. \quad (3.31)$$

This coincides with the result for σ_Q obtained earlier in [5, 25] to *leading order* in B . Our result (3.31), however, is valid to *all orders* in B . Our calculation provides a rigorous proof that the conductivity coefficient does not depend on the magnetic field.

4 The Fate of the Shear Modes and Charge Diffusion

In this section, we will consider the fluctuations (2.39) describing the shear channel. Let us introduce

$$\begin{aligned} h_{tx} &= e^{-i\omega t + i q y} c_2(r)^2 H_{tx}, \\ h_{xy} &= e^{-i\omega t + i q y} c_2(r)^2 H_{xy}, \\ a_t &= i e^{-i\omega t + i q y} \hat{a}_t, \\ a_y &= i e^{-i\omega t + i q y} \hat{a}_y, \end{aligned} \quad (4.1)$$

where $H_{tx}, H_{xy}, \hat{a}_t$ and \hat{a}_y are functions of the radial coordinate r . Expanding eqs. (2.23) to linear order we obtain the following system of equations

$$\begin{aligned} 0 &= H''_{tx} + \left[\ln \frac{c_2^4}{c_1 c_3} \right]' H'_{tx} - \frac{c_3^2}{c_2^4} (4h^2 \alpha^4 + q^2 c_2^2) H_{tx} - \frac{q \omega c_3^2}{c_2^2} H_{xy} \\ &\quad - \frac{4h \alpha^2 q c_3^2}{c_2^4} \hat{a}_t - \frac{4h \alpha^2 \omega c_3^2}{c_2^4} \hat{a}_y, \end{aligned} \quad (4.2)$$

$$0 = H''_{xy} + \left[\ln \frac{c_1 c_2^2}{c_3} \right]' H'_{xy} + \frac{\omega^2 c_3^2}{c_1^2} H_{xy} + \frac{q \omega c_3^2}{c_1^2} H_{tx}, \quad (4.3)$$

$$0 = \hat{a}_t'' + \left[\ln \frac{c_2^2}{c_1 c_3} \right]' \hat{a}_t' - \frac{q^2 c_3^2}{c_2^2} \hat{a}_t - \frac{q \omega c_3^2}{c_2^2} \hat{a}_y - \frac{h \alpha^2 q c_3^2}{c_2^2} H_{tx}, \quad (4.4)$$

$$0 = \hat{a}_y'' + \left[\ln \frac{c_1}{c_3} \right]' \hat{a}_y' + \frac{\omega^2 c_3^2}{c_1^2} \hat{a}_y + \frac{q \omega c_3^2}{c_1^2} \hat{a}_t + \frac{h \alpha^2 \omega c_3^2}{c_1^2} H_{tx}. \quad (4.5)$$

In addition, we have two first class constraints obtained from varying the action with respect to the gauge fixed components h_{xr} and a_r

$$0 = \frac{q}{2}H'_{xy} + \frac{\omega c_2^2}{c_1^2}H'_{tx} + \frac{2h\alpha^2}{c_2^2}\hat{a}'_y, \quad (4.6)$$

and

$$0 = \frac{q}{c_2^2}\hat{a}'_y + \frac{\omega}{c_1^2}\hat{a}'_t. \quad (4.7)$$

If we set $h = 0$ we see that the equations describing (H_{tx}, H_{xy}) and (\hat{a}_t, \hat{a}_y) decouple from each other. In this case, the equations for (H_{tx}, H_{xy}) describe the shear modes with dispersion relation (2.14) [26]. Similarly, the equations for (\hat{a}_t, \hat{a}_y) describe diffusion with dispersion relation (2.15) [26].

To continue, we introduce quasinormal modes invariant under the residual diffeomorphisms

$$\begin{aligned} Z_H &= qH_{tx} + \omega H_{xy}, \\ Z_A &= q\hat{a}_t + \omega\hat{a}_y - \frac{\omega}{q}h\alpha^2 H_{xy}. \end{aligned} \quad (4.8)$$

Then from eqs. (4.2)-(4.8) we obtain two decoupled gauge invariant second order equations for Z_H and Z_A of the form (3.11), (3.12). These equations are rather lengthy and we will not write them in the paper. Below, we will present them to the lowest orders in ω and q . The quasinormal modes Z_H and Z_A have the same boundary conditions as discussed in the previous section. Namely, Z_H and Z_A are incoming waves at the horizon $r = 1$ and vanish on the boundary $r = 0$. Repeating the same analysis as in the previous section, we arrive at the following ansatz for our solution

$$\begin{aligned} Z_H(r) &= f(r)^{-i\mathfrak{w}/2} \frac{\mathfrak{q}}{h} \left(F_1(r) + \mathfrak{q}^2 F_3(r) + \frac{\mathfrak{q}^4}{h^2} F_5(r) + \mathcal{O}(\mathfrak{q}^6) \right), \\ Z_A(r) &= f(r)^{-i\mathfrak{w}/2} \left(F_2(r) + \mathfrak{q}^2 F_4(r) + \frac{\mathfrak{q}^4}{h^2} F_6(r) + \mathcal{O}(\mathfrak{q}^6) \right), \end{aligned} \quad (4.9)$$

where

$$\mathfrak{w} = \frac{\omega}{2\pi T}, \quad \mathfrak{q} = \frac{q}{2\pi T}, \quad (4.10)$$

and the functions $F_i(r)$ are non-singular and non-vanishing at the horizon and have a perturbative expansion in h . In addition, they satisfy the Dirichlet conditions on the boundary. Since the equations are homogeneous, we can choose the following normalization at the horizon $r = 1$

$$F_2(r)|_{r=1} = 1, \quad F_4(r)|_{r=1} = F_6(r)|_{r=1} = \dots = 0. \quad (4.11)$$

Once (4.11) is chosen, no further normalization condition for F_1, F_3, F_5, \dots at $r = 1$ can be imposed. We parameterize the dispersion relation as follows

$$\mathfrak{w} = -i \frac{\mathfrak{q}^4}{h^2} C, \quad (4.12)$$

where the diffusion constant C is assumed to have a perturbative expansion in h . As in the previous section, we will first solve the equations to leading order in h , that is ignoring the h dependence in C and $F_i(r)$. Then we will generalize our method for arbitrary h and find C to all orders in h . To leading order in \mathfrak{q} we obtain the following simple equations

$$F_1'' - \frac{2}{r} F_1' = 0, \quad (4.13)$$

and

$$F_2'' = 0. \quad (4.14)$$

The solution with the prescribed above boundary conditions is

$$F_1(r) = b_1 r^3, \quad F_2(r) = r. \quad (4.15)$$

The integration constant b_1 so far is not fixed. Our analysis shows that it is fixed by requiring that the function $F_5(r)$ is non-singular at the horizon. We will not present the details of this analysis since they are not important for our purposes. At the next order in \mathfrak{q} we obtain equations for F_3 and F_4 . They can be solved analytically but these solutions are not of interest for us. It turns out that the diffusion constant C is determined from the equation for F_6 which reads

$$F_6'' + \frac{16C^2 r^2}{9(1-r^3)} F_2'' - \frac{Cr(27r(-1+r^3) + 16C(2+r^3))}{9(-1+r^3)^2} F_2' + \frac{3Cr(2+r^2)}{2(-1+r^3)^2} F_2 = 0. \quad (4.16)$$

Note that to determine F_6 we need to know only F_2 which was found in (4.15). The other functions F_1, F_3, F_4 do not enter this equation. It is possible to find the general solution for F_6 analytically. We will not write it because it is rather lengthy. It turns out that to determine C it is enough to study the behavior of F_6 near the horizon. We find that near the horizon F_6 has a logarithmic singularity of the form

$$F_6(r) = \frac{C}{54} (27 - 32C) \ln(1-r) + \dots, \quad (4.17)$$

where the ellipsis stands for the non-singular terms. Since F_6 has to be smooth at the horizon it follows that we can have a non-trivial solution

$$C = \frac{27}{32} + \mathcal{O}(h). \quad (4.18)$$

The two integration constants of F_6 are fixed by requiring that F_6 vanishes at $r = 0$ and $r = 1$.

Now it is clear how to generalize the above procedure for arbitrary h . The diffusion constant C is obtained from requiring that the solution does not have a logarithmic singularity at the horizon. In fact, it is possible to carry out the near-horizon analysis for arbitrary h . Repeating similar steps as in Appendix A, we obtain the final result

$$C = \frac{(3 - h^2)^3}{32}. \quad (4.19)$$

Thus, we have managed to reproduce the subdiffusive mode (2.18) in the shear channel on the gravity side! We proved that it does exist and found the value of the diffusive constant C to *all orders* in magnetic field in the large N limit.

To summarize, we have obtained the supergravity solution with the following dispersion relation

$$\mathfrak{w} = -i \frac{\mathfrak{q}^4 (3 - h^2)^3}{h^2 32}. \quad (4.20)$$

Let us compare it with the field theory counterpart (2.18). Using eqs. (4.10), (2.27) and (2.35) we can rewrite (2.18) in the form

$$\mathfrak{w} = -i \frac{\mathfrak{q}^4 (3 - h^2)^3}{h^2 8\alpha^2} \eta g^2. \quad (4.21)$$

The factor $(3 - h^2)^3$ comes from rewriting the temperature in terms of α and h . Comparing (4.20) and (4.21) we conclude that

$$\eta = \frac{\alpha^2}{4g^2}, \quad (4.22)$$

where, to recall, α is related to the temperature and the magnetic field as follows

$$T = \frac{\alpha}{4\pi} \left(3 - \frac{B^2}{\alpha^4} \right). \quad (4.23)$$

Note that despite the simple form (4.22), η , after being rewritten in terms of T and B , has a non-trivial dependence on the magnetic field. Recalling the expression for the entropy density (2.30) we, finally, obtain

$$\frac{\eta}{s} = \frac{1}{4\pi}. \quad (4.24)$$

We see that the ratio η/s saturates the KSS bound [7]. This extends the universality theorem of [9] for the case of strongly coupled plasma in external magnetic field.

5 The Kubo Formula

In the previous section, we extracted the ratio of the shear viscosity to the entropy ratio from the dispersion relation of the shear quasinormal modes. Alternatively, one can use a Kubo formula, as in [9], to directly evaluate the shear viscosity:

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega i} [G_{xy,xy}^A(\omega, 0) - G_{xy,xy}^R(\omega, 0)] , \quad (5.1)$$

where $G_{xy,xy}^A$ ($G_{xy,xy}^R$) is the advanced (retarded) two-point correlation function of the stress-energy tensor (with indicated spatial indices) evaluated at zero momentum. Naively, the universality arguments presented in [9] do not hold here. Indeed, the dual gravitational mode, h_{xy} does not generically decouple as one can see from eqs. (4.2)-(4.5). Furthermore, the background of the bulk vector field (holographically dual to the background boundary magnetic field) is not solely polarized along the time direction, as assumed in [9]. In this section, we revisit the argument of [9] and extend the universality theorem to the case of background magnetic field in (2+1)-dimensional strongly coupled plasma.

First, notice that even though the gravitational mode h_{xy} does not decouple for non-zero momentum q , see eqs. (4.2)-(4.5), the decoupling occurs for $q = 0$, which is all what is needed for the computation of the correlation functions in (5.1). Physically, the reason why such a decoupling occurs is because for vanishing q there is an additional symmetry in plasma associated with a reflection along the y -axis. As a result, the graviton polarization h_{xy} is the only fluctuating mode which is doubly parity odd under reflections along the x - and y -axis. Hence, it must decouple. Second, from (4.3) we see that the rescaled (see eq. (4.1)) graviton wavefunction H_{xy} at $q = 0$ satisfied the equation of motion for the minimally coupled massless scalar in a regular Schwarzschild horizon geometry (2.24). But now, we are precisely in the setup of the universality arguments of [9]! One can literally repeat the analysis presented there to establish that the shear viscosity η , as determined by (5.1), is proportional to the entropy density s with the proportionality coefficient as in (4.24).

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A Exact value of C

For a background magnetic field h held fixed in the hydrodynamic limit, the sound channel dispersion relation is given by (3.17), with the constant $C \equiv C(h)$. Here, we outline steps necessary to obtain exact analytical expression for $C(h)$.

Within the ansatz (3.18), F_1 is given by (3.20). From (3.11) and (3.12) we further find the following equations for F_2 and F_3

$$0 = F_2'' + \frac{2r^4h^2 - r^3(h^2 + 1) - 2}{r(r^3h^2 - r^2 - r - 1)(r - 1)}F_2' - \frac{(h^2 - 3)\alpha C^2}{8r(r^3h^2 - r^2 - r - 1)(r - 1)}, \quad (\text{A.1})$$

$$\begin{aligned} 0 = & F_3'' - 6\frac{r^4h^2 - 1}{r(r^4h^2 - 3)}F_3' + \frac{12r^2h^2}{r^4h^2 - 3}F_3 \\ & - \frac{r^2(h^2 - 3)(r^3(h^2 + 1) - 4)(r^4h^4 + r^4h^2 - 16h^2r + 9h^2 + 9)}{2C^2\alpha(r^3h^2 - r^2 - r - 1)(r - 1)(r^4h^2 - 3)}F_2' \\ & - \frac{24r^2(h^2 - 3)h^2}{C^2\alpha(r^4h^2 - 3)}F_2 + \mathcal{J}_3, \end{aligned} \quad (\text{A.2})$$

where

$$\begin{aligned} \mathcal{J}_3 = & \frac{r^2}{32(r - 1)^2(r^3h^2 - r^2 - r - 1)^2(r^4h^2 - 3)} \left(512r^3(r - 1)(r^3h^2 - r^2 - r - 1)C^2 \right. \\ & + 16r^2(-54(h^2 + 1) + 84h^2r + 27(h^2 + 1)^2r^3 - 60h^2(h^2 + 1)r^4 + 24h^4r^5 \\ & + 3h^2(h^2 + 1)^2r^7 - 6h^4(h^2 + 1)r^8 + 4r^9h^6)C - (h^2 - 3)^2(-36(h^2 + 1) + 40h^2r \\ & + 9(h^2 + 1)^2r^3 - 32h^2(h^2 + 1)r^4 + 48h^4r^5 + h^2(h^2 + 1)^2r^7 - 12h^4(h^2 + 1)r^8 \\ & \left. + 8r^9h^6) \right). \end{aligned} \quad (\text{A.3})$$

It is straightforward to construct power series solutions first for F_2 (A.1) and then for F_3 (A.2) near the horizon. Regularity of $\{F_2, F_3\}$ for small $x = 1 - r$ then uniquely fixes C as in (3.28).

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